

Discrete Mathematics and Logic

Build precise reasoning with propositions, sets, and relations.

Beginner lesson · Estimated reading time: 8–12 minutes

Introduction

Discrete mathematics studies structures made of distinct objects. Logic helps express and test claims; sets organize collections; relations describe connections between objects. These ideas underpin algorithms, databases, digital circuits, and formal verification.

Learning objectives

- Distinguish propositions from questions or commands.
- Evaluate NOT, AND, OR, and implication.
- Use truth tables to test logical statements.
- Apply basic set notation and understand relations.

1. Propositions and truth values

A proposition is a declarative statement that is either true or false in a given interpretation. “7 is prime” is true; “10 < 3” is false. “Close the door” is a command, not a proposition. A variable such as P can stand for a proposition.

Negation NOT P reverses the truth value of P . If P is true, $\neg P$ is false; if P is false, $\neg P$ is true.

2. Logical connectives

Conjunction $P \wedge Q$ (AND) is true only when both P and Q are true. Disjunction $P \vee Q$ (OR) is true when at least one is true, including when both are true. Negation $\neg P$ reverses P .

Implication $P \rightarrow Q$ is false only when P is true and Q is false. Read it as “if P , then Q .” If P is false, the implication is true in classical logic, regardless of Q .

3. Truth tables

A truth table lists every possible truth-value combination. For two propositions P and Q , there are $2^2 = 4$ rows.

$P \mid Q \mid P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid \neg P \mid \neg Q \mid P \leftrightarrow Q$

To compare two expressions, evaluate both columns row by row. If they match on every row, the expressions are logically equivalent.

4. Sets and set operations

A set is a collection of distinct elements. If $A = \{1, 2, 3\}$ and $B = \{3, 4\}$, then $A \cup B = \{1, 2, 3, 4\}$ (union), $A \cap B = \{3\}$ (intersection), and $A \setminus B = \{1, 2\}$ (difference). The order of elements does not matter, and repeated elements do not create new members.

Use $x \in A$ to mean “ x is an element of A ” and $A \subseteq B$ to mean every element of A is also in B . The empty set $\emptyset$ contains no elements.

5. Relations and applications

A relation from A to B is a set of ordered pairs drawn from $A \times B$. For example, “is less than” relates some numbers to others. Relations can be represented as tables, pairs, or graphs. In databases, a relation-like table organizes records by fields.

A relation on one set may be reflexive (each element relates to itself), symmetric (aRb implies bRa), or transitive (aRb and bRc imply aRc). Check the definition carefully before assigning a property.

6. Common mistakes

- Treating logical OR as exclusive OR when the context uses inclusive OR.
- Assuming $P \rightarrow Q$ is the same as $Q \rightarrow P$.
- Confusing an element $x \in A$ with a subset $A \subseteq B$.
- Counting duplicate set elements more than once.
- Declaring a relation symmetric or transitive from only one example.

Practice exercises

EX1: If P is false and Q is true, evaluate $P \wedge Q$, $P \vee Q$, and $P \rightarrow Q$.

Answer: $P \wedge Q$ is false; $P \vee Q$ is true; $P \rightarrow Q$ is true because a false antecedent makes the implication true in classical logic.

EX2: Let $A = \{a, b, c\}$ and $B = \{b, c, d\}$. Find $A \cap B$ and $A \cup B$.

Answer: $A \cap B = \{b, c\}$; $A \cup B = \{a, b, c, d\}$.

EX3: Is the implication $P \rightarrow Q$ always equivalent to $Q \rightarrow P$?

Answer: No. For P true and Q false, $P \rightarrow Q$ is false while $Q \rightarrow P$ is true.